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2004 J. Phys.: Condens. Matter 16 4941

(http://iopscience.iop.org/0953-8984/16/28/014)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **16** (2004) 4941–4953 PII: S0953-8984(04)76760-0

Magnetophonon resonance in quantum wells in tilted field: what is concealed behind its angular dependence?

V V Afonin[1,2], V L Gurevich[1,2], R Laiho[1], M O Safonchik[1,2], M A Shakhov[1,2] and M L Shubnikov[1,2]

[1] Wihuri Physical Laboratory, University of Turku, FIN-20014 Turku, Finland
[2] A F Ioffe Institute, 194021 Saint Petersburg, Russia

Received 19 February 2004
Published 2 July 2004
Online at stacks.iop.org/JPhysCM/16/4941
doi:10.1088/0953-8984/16/28/014

Abstract

Magnetophonon resonance in quantum wells in a tilted magnetic field **B** is investigated. Measurements of the Hall coefficient and correspondingly of the carrier concentration as a functions of magnetic field and temperature are simultaneously performed. It is shown that the experimental data can be interpreted in terms of a great sensitivity to the effects of varying the two dimensional carrier concentration n_s in a certain concentration interval. In other words, the observed angular dependence of the MPR amplitudes is a manifestation of dependence of n_s on the magnitude of the magnetic field B. We believe that such a dependence can be relevant in general for the interpretation of magnetotransport in nondegenerate 2D electron gas.

1. Introduction

Magnetophonon resonance (MPR) in semiconductors is reached every time the limiting frequency of a longitudinal optic phonon ω_0 equals the cyclotron frequency of an electron, Ω, times some small integer, $\mathcal{N}$ (see [1, 2]). Along with cyclotron resonance, it has become one of the main instruments of semiconducting compound spectroscopy.

The advances in semiconductor nanofabrication in recent years have made available nanostructures of great crystalline perfection and purity. The electrical conduction and some other transport phenomena in such specimens has been the focus of numerous investigations, both theoretical and experimental. In particular, the discovery of MPR in the quantum wells took place in the pioneering paper by Tsui *et al* [3]. The most detailed experimental investigation of MPR in quantum wells has been done by Nicholas and co-workers (see [4] and the references therein). It has been shown that there is a qualitative difference between MPR in 2D and 3D structures.

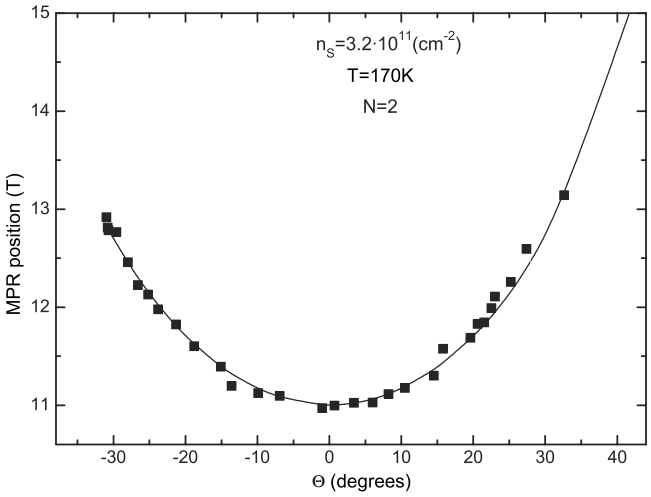

Figure 1. MPR maximum position as a function of the angle of tilting. The line corresponds to the dependence $B_2(0)/\cos\theta$ ($n_s = 3.2 \times 10^{11}$ cm^{-2}).

In the 2D case MPR can exist only in a relatively narrow interval of electron concentrations n_s. This has been indicated in [4] and the physics of this phenomenon has been described in [5]. In a special group of experiments [3, 4] an angular dependence of MPR has been investigated. As is well known, the 2D magnetoconductance, including the MPR [6], at high magnetic fields B should depend on the combination $B\cos\theta$ (see, for example, [7]). Here θ is the angle between the magnetic field **B** and the perpendicular to the plane of the well. One can easily understand this using the following classical analogy. In the 2D case the curvature of an electron's trajectory (in the course of electron's periodic motion in the plane) can be considered as nonexistent in the direction perpendicular to the plane because of the electron's interaction with the walls of the well. It means that all the physical quantities can depend only on the perpendicular component of the field. In particular, the position of the $\mathcal{N}$th MPR is given by

$$B_{\mathcal{N}}(\theta)\cos\theta = B_{\mathcal{N}}(0) \tag{1}$$

where $B_{\mathcal{N}}(\theta)$ is the position of the MPR maximum for **B** directed at the angle θ to the perpendicular to the plane of the well while

$$B_{\mathcal{N}}(0) = m\omega_0 c/e\mathcal{N}.$$

Here ω_0 is the limiting frequency of the optic phonons (we will not discriminate between the longitudinal frequency ω_l and the transverse one ω_t, because of the insufficient accuracy of our experiment) and m is the effective mass. Experimentally the angular dependence has been investigated by Tsui *et al* [3] and Brummel *et al* [8]. They have observed the angular dependence of the amplitude of the MPR maximum that appeared to be very sharp, whereas according to equation (1) the amplitude of the MPR maximum should be independent of θ. This gives a drastic disagreement between the experiment and theory, and means that there is some feature in the system considered depending on the total magnetic field B rather than on the combination $B\cos\theta$. The position of the MPR maximum in the angular interval $\theta < 30°$ (see figure 1) is well described by equation (1). It means that in this interval one observes a two-dimensional situation with an angular dependence of an MPR maximum. At larger angles the shape of angular dependence of the MPR positions in the present experiment is changed. We will not discuss in the present paper angles bigger than 25°.

One of the main characteristics of the sample is the carrier concentration n_s. It is usually implied that it depends neither on the temperature nor on the magnetic field. As a rule, this is true at low temperatures where most experiments with nanostructures are performed. However, the MPR experiments are made at relatively high temperatures, the highest amplitudes in GaAs being observed at T about 180 K (they depend on $\mathcal{N}$ but only slightly). It is natural therefore to check the temperature and magnetic field dependence of the concentration. In order to control the electron concentration n_s, we have performed observations of MPR along with measurements of the Hall effect in 2D structures. Thus the purpose of the present paper is the investigation of the MPR, simultaneous measurement of the magnetic field and the temperature dependence of n_s and interpretation of the obtained data on MPR based on the data on n_s. We will see that the MPR oscillation is very sensitive to the electron concentration variation. We would like to emphasize that other magnetotransport phenomena at sufficiently high temperatures may also be sensitive to the variation of n_s.

2. Experimental details

Three series of GaAs/Al_xGa_{1-x}As quantum well samples grown by molecular beam epitaxy were cut into the shape of a typical Hall bar for observation of the Shubnikov–de Haas (SdH) and the MPR oscillations. To avoid overheating of the sample during the magnetic field pulse, we chose the measuring current to be sufficiently small (of the order of 5 μA). The measurements were carried out over the temperature interval of 4.2–300 K in pulsed magnetic fields up to $B = 40$ T with the pulse duration of 8 ms. The main tool for collecting the data in our pulsed field installation is the data acquisition card with four fast independent 1 μs, 12 bit digital channels having 128 kb buffer memory each.

The measured signal has a smooth nonlinear component with the amplitude much bigger than the amplitude of the investigated MPR oscillation. To single out the oscillation and to get rid of the high frequency noise we have used a software package based on the approximation of the curve by the polynomial minimum squares method with the Gaussian weight function. The method permits one to process the signals properly, particularly at the edges of the interval of magnetic field variation. However, it brings about some distortion of the form of oscillation, especially for the peaks near the edges (namely, the oscillation shifts towards smaller fields while its amplitude goes down). Nevertheless, if an edge of the interval is within the same phase of the MPR, the distortion of the last peak should be also the same for all the curves and the results can be compared. One of the plots obtained as a result of such a treatment is shown in figure 2. As under rotation of the specimen the maxima shift towards bigger fields (see equation (1)), the maximal pulse field B_{max} should also have the angular dependence $B_{max}/\cos\theta$. All the remaining parameters used for the processing stay the same for all the pulses. The dots obtained in such a manner are indicated in figures 1–8. To draw the lines through these dots we have used the program based on the method of weighted minimal squares.

Well developed SdH oscillations periodic in $1/B$ were observed at $T = 4.2$ K and used to determine the values of the low-temperature carrier concentrations of the samples, namely $n_s = 2.2$, 3.2 and 4.0×10^{11} cm^{-2}. As pronounced MPR oscillations require sufficient optical phonon population they are usually observable at elevated temperatures. For this reason, we applied the Hall geometry to investigate the dependence of the carrier concentration on the applied field at fixed temperatures between 80 and 300 K. Correspondingly, the temperature dependence of n_s was determined between 80 and 300 K for B between 0.95 and 27 T (see figures 3 and 4).

The MPR oscillations, also periodic in $1/B$, were recorded at different temperatures in the range $T = 170$–230 K. In these measurements the magnetic field was tilted at an angle θ.

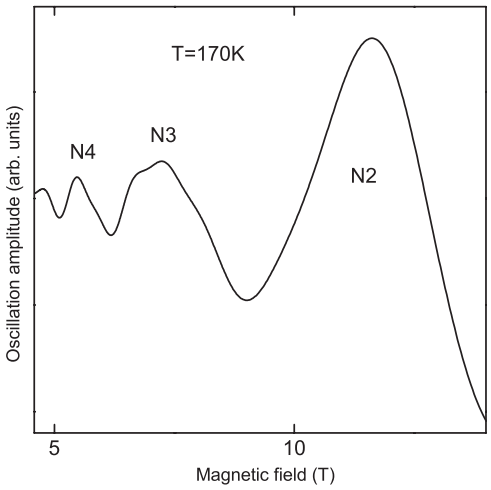

Figure 2. The positions of the MPR maxima for $T = 170$ K and $n_s = 4.0 \times 10^{11}$ cm^{-2}.

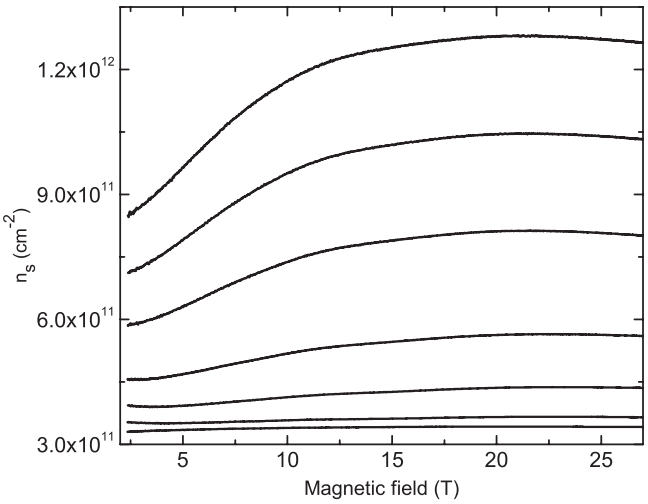

Figure 3. Variation of 2D electron concentration n_s for various temperatures (in kelvin) calculated from experimental Hall data when the magnetic field B is continuously swept within the indicated interval. The data are for the sample with the low temperature concentration $n_s = 3.2 \times 10^{11}$ cm^{-2}. The temperatures from the bottom to the top are 20, 140, 170, 200, 240, 270 and 300 K.

From the data the amplitude and the field positions of the MPR oscillation peaks with $\mathcal{N} = 2$, 3 and 4 were analysed (for $\mathcal{N} = 2$ and 3 they are given in figures 5–8). Depending on the sample and the temperature, the accuracy of the results varied between 2% and 5%.

The magnetic field interval 6–15 T is important for our purpose. However, we have also observed the Hall resistivity ρ_{xy} outside this interval, namely down to the field 0.8 T. We have done this in order to make estimates of τ at the temperatures we are interested in. Such estimates are possible because provided the time of relaxation τ is energy dependent, in general, the magnetic field dependence of ρ_{xy} is described not by one but by two straight lines, and by writing about bend we mean the transition from one straight line to another in the region of low magnetic fields where $\Omega\tau$ is of the order of 1. The origin of this bend is

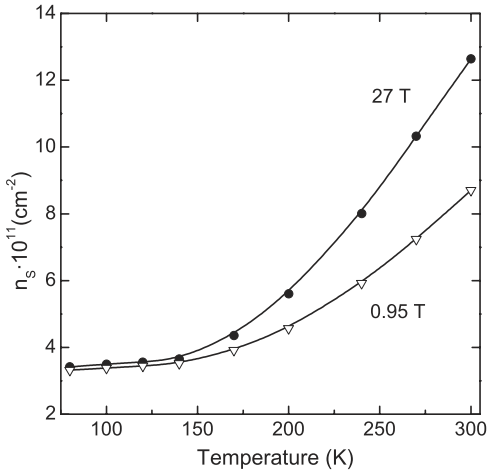

Figure 4. Variation of 2D electron concentration n_s as a function of temperature T. The data are for the sample with the low temperature concentration $n_s = 3.2 \times 10^{11}$ cm^{-2}.

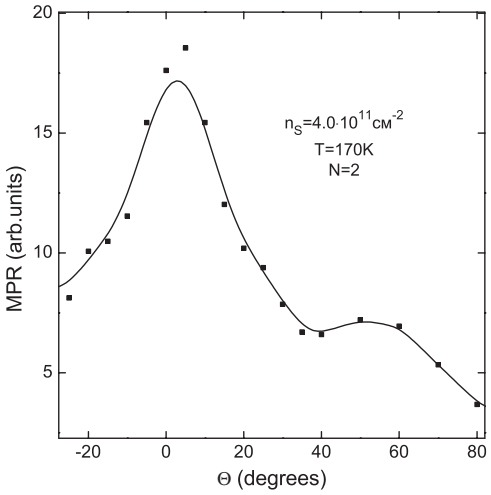

Figure 5. Angular dependence of the height of the MPR maximum; $\mathcal{N} = 2$, $n_s = 4.0 \times 10^{11}$ cm^{-2}, $T = 170$ K.

well known (for instance [13]). We have not seen any bend in the linear dependence of ρ_{xy} on B *in the region of low magnetic fields*, although it should be seen provided $\Omega\tau = uB/c$ is of the order of 1 (here u is the electron mobility) and τ is energy dependent. This means that to reach the region $\Omega\tau \approx 1$ one should make the magnetic field even lower. This gives an upper bound for τ in the relevant temperature region.

This shows (see figure 3) that for $B \gtrsim 6$ T the variation of concentration n_s as a function of magnetic field B is rather big and the deviation from the linear dependence of $\rho_{xy}(B)$ in the region of the strong magnetic fields $uB/c \gg 1$ we are interested in cannot be explained as originating from the difference between the resistivity

$$\rho_{xy} = \frac{\sigma_{yx}}{\sigma_{yx}^2 + \sigma_{xx}^2}$$

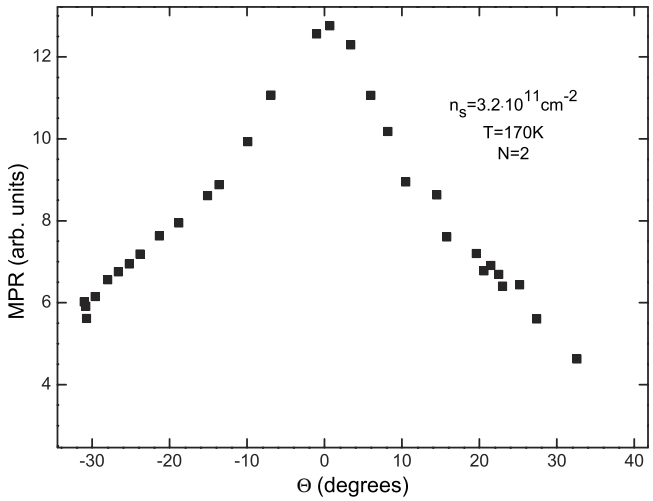

Figure 6. Angular dependence of the MPR maxima for $n_s = 3.2 \times 10^{11}$ cm^{-2} (a more rough algorithm of the oscillation separation).

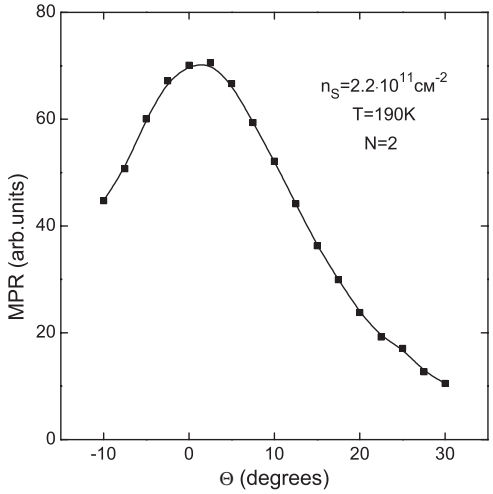

Figure 7. Angular dependence of the height of the MPR maximum; $\mathcal{N} = 2$, $n_s = 2.2 \times 10^{11}$ cm^{-2}, $T = 190$ K.

and the reciprocal conductivity $1/\sigma_{yx}$. Indeed, in this region of magnetic field

$$\sigma_{xx}^2 \approx \sigma_{xy}^2 (\Omega\tau)^{-2},$$

so that the difference between ρ_{xy} and $1/\sigma_{xy}$ is proportional to the small parameter $(\Omega\tau)^{-2}$. If one assumes that $\Omega\tau \approx 1$ for $B = 0.5$ T then the difference $\rho_{xy} - 1/\sigma_{xy}$ in the region of the second MPR maximum (11 T) would be not bigger than 0.25%. However, we observe a nonlinear deviation of the order of several per cent.

In the relevant magnetic field interval 6–12 T, the rate of the electron concentration variation is temperature dependent (see figure 3). It is about 1.5% T^{-1} at 170 K and 3% T^{-1} at 200 K. From the low temperatures up to 140 K there is no noticeable magnetic field dependence of concentration. This behaviour fully correlates with the temperature dependence of the Hall

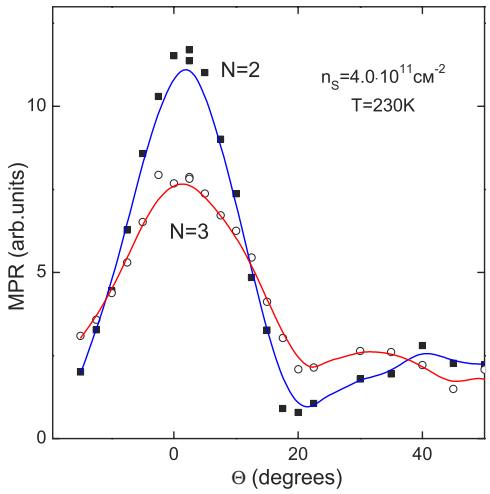

Figure 8. Angular dependence of the height of the MPR maxima; $\mathcal{N} = 2$ and 3, $n_s = 4.0 \times 10^{11}$ cm^{-2}, $T = 230$ K.

(This figure is in colour only in the electronic version)

coefficient (see figure 4). One can see a rather strong temperature dependence from 140 K. Note that when either the temperature or magnetic field goes up, the concentration n_s also goes up.

3. Experimental results and their theoretical description

As one can expect, the relative rate of concentration variation with the temperature (see figure 4) is bigger than with magnetic field. The latter, however, is by a factor of 5–6 bigger as compared with the concentration variation associated with the spin magnetic moment of the free electrons. The well-known corrections to the g-factor of electrons in quantum wells [9] can be disregarded as they also depend only on the combination $B\cos\theta$.[3] In general, all the effects originating from the 2D electrons in a quantum well depend on B and θ only in the combination $B\cos\theta$, and therefore cannot explain the angular dependence of MPR.

This makes one think of the electrons that tunnel from centres outside the well into the quantum well. Usually the electron levels in such centres go down with the magnetic field B. Indeed, as B goes up the electron wavefunctions become closer to the nuclei and therefore their binding becomes tighter, or, in other words, the absolute value of the electron binding energy goes up. As a result, the level should go down as well as the electron concentration n_s in the well. In our experiment n_s has an opposite behaviour.

Another peculiarity of the variation of n_s that cannot be explained by a simple variation of the donor levels as a function of B is saturation of the function $n_s(B)$ for relatively small value of B (see figure 3). It begins for $B \approx 12$ T. It is interesting to note that for these values of B the Bohr radius $a_B = \hbar^2/me^2$ becomes of the order of magnetic length $a_L = \sqrt{\hbar c/eB}$.

Our experiment shows that n_s depends on B for $a_B > a_L$ and this dependence rapidly disappears in the opposite case. For instance, this may indicate the existence of a weakly bound state that is destroyed by the magnetic field. Such a state would be charged and therefore it could not exist within the well (otherwise the condition $\Omega\tau \gg 1$ would be violated because

[3] Note that *any* magnetic field dependent corrections to the 2D electron Hamiltonian enter in the combination $B\cos\theta$.

of strong electron scattering). Thus the electron concentration n_s should depend on the total magnetic field rather than its perpendicular component.

Actually, in the situation considered in the present paper it is difficult to indicate a definite mechanism responsible for the temperature and the magnetic field dependence of the electron concentration n_s in the conduction band of the quantum wells at relatively high temperatures (the Boltzmann statistics). The quantum wells under consideration are in fact multilayer structures that may contain donors and acceptors of various types and in various positions outside the well. They determine the electron concentration n_s within the well. As they are *in the bulk of the sample* outside the well n_s can depend on the value of the magnetic field B *but not on its direction*. The dependence $n_s(B)$ we have observed may be considered as rather unusual; to determine its origin is an interesting physical problem that we hope to investigate more carefully in the future. We would like to emphasize once again that this problem is of substantial interest and warrants further investigation.

Our purpose is to prove that the dependence $n_s(B)$ we have observed is sufficient to interpret the angular dependence of the MPR maxima investigated in detail by Nicholas *et al* (see the review paper [4] and the references therein). Note that in the same paper [4] a very strong dependence of the MPR amplitude is demonstrated in the region of high electron concentrations. The MPR amplitude starts to go down at $n_s \approx 10^{11}$ cm^{-2}. At $n_s = 3 \times 10^{11}$ cm^{-2} the decrease becomes extremely strong. The amplitude decreases by a factor of 12 under the increase of concentration from $n_s = 3 \times 10^{11}$ to 5.5×10^{11} cm^{-2}. Thus the dependence of the MPR maxima on n_s should take place *irrespective of the mechanism of variation of* n_s *itself*.

Such a dependence of the MPR amplitude on concentration is surprising. Usually the role of coulomb e–e interaction is determined by the so-called gas parameter η given by

$$\eta = e^2 n^{1/2} / \varepsilon k_B T. \tag{2}$$

This is the ratio of the energy of Coulomb e–e interaction to the kinetic energy of electrons. η is of the order of $1/4$ for $n_s = 10^{11}$ cm^{-2} and $T = 200$ K, i.e. it seems that one can neglect the e–e interaction.

Another parameter important for our problem is the degeneracy parameter, i.e. the ratio

$$|\mu| / k_B T \tag{3}$$

of the modulus of chemical potential μ to $k_B T$. It becomes of the order of or smaller than 1 at $n_s \approx 3 \times 10^{11}$ cm^{-2}—see [4]. Physically, this means that under these conditions the electron system is near the borderline between the Boltzmann and the Fermi statistics. However, the onset of degeneracy normally changes the effect by something like a factor of 2 or so, whereas a much more substantial variation of the MPR amplitude with the electron concentration has actually been observed.

The physics of such a behaviour has been described in [5]. Qualitatively it can be interpreted in the following way. Usually one interprets MPR as a result of electron transitions between two Landau levels. However another, a less direct approach is also possible. One can treat MPR as an enhancement of the interaction of a pair of electrons due to the exchange of an optic phonon (a pole of the scattering amplitude). One should, however, take into consideration that, apart from the interaction due to exchange of a phonon, the electrons have also a direct Coulomb interaction. The sum of these two interactions can be described by a potential [10] for which the Fourier component is given by

$$V = \frac{2\pi e^2}{q\varepsilon(\omega)} \tag{4}$$

where q is the absolute value of the Fourier variable while

$$\varepsilon(\omega) = \varepsilon_\infty \frac{\omega_l^2 - (\omega + i\Gamma)^2}{\omega_t^2 - (\omega + i\Gamma)^2}. \tag{5}$$

Here ε_∞ is the lattice dielectric susceptibility for $\omega \to \infty$ while Γ is the phonon damping due to the phonon anharmonicity.

Equation (5) describes the direct interaction between two electrons. One should, however, also allow for the indirect interaction where the first electron interacts with the next (intermediate) one and this, in turn, interacts with another electron, etc (see the detailed derivation in [5, 6]). Taking this in consideration, one should take into account the following two points. First, one can consider any electron as the intermediate one. In other words, one should sum over all the conduction electrons. This will give the factor n_s.

Second, the interaction we are discussing is of a resonant nature. In the 2D case, the electron spectrum, unlike the 3D case, has no component of the quasimomentum along the magnetic field. As a result, the characteristic time of the e–e interaction is not $m/\hbar q_z^2$ as in the 3D case but is determined by $1/\Gamma_e$ where Γ_e is the electron damping. This means that the electrons will be in resonance during the time of the of order of $1/\Gamma_e$. As a result, we get for the indirect interaction equation (4) with an extra factor [6]

$$\frac{2\pi e^2}{q\varepsilon(\omega)} \frac{n_s}{\hbar(\omega - \mathcal{N}\Omega\cos\theta + i\Gamma_e)}.$$

The whole expression is dimensionless.

Now we should take into account that this interaction may take place $1, 2, 3, \ldots$ times. As a result, the full interaction is

$$V_{\text{full}} = \frac{2\pi e^2}{q\varepsilon(\omega)} \left[1 - \frac{n_s}{\hbar(\omega - \mathcal{N}\Omega\cos\theta + i\Gamma_e)} \frac{2\pi e^2}{q\varepsilon(\omega)} \right]^{-1}. \tag{6}$$

The interaction becomes very strong provided the expression in the square brackets vanishes. In fact this condition means the existence of *electron transitions between Landau levels due to the interaction with the mixed electron–phonon mode.*

We will solve the equation

$$1 - \frac{n_s}{\hbar(\omega - \mathcal{N}\Omega\cos\theta + i\Gamma_e)} \frac{2\pi e^2}{q\varepsilon(\omega)} = 0 \tag{7}$$

by iterations considering the damping as relatively small. In the lowest approximation we have two solutions we are interested in, namely

$$\omega = \mathcal{N}\Omega\cos\theta, \qquad \text{and} \qquad \omega = \omega_t.$$

This means that

$$\omega_t = \mathcal{N}\Omega\cos\theta. \tag{8}$$

This condition determines the MPR peak positions. The next, imaginary, approximation determines the width of the $\mathcal{N}$th MPR peak

$$\Gamma_{\mathcal{N}} = \Gamma_e + \frac{n_s}{n_{up}}\Omega\cos\theta, \tag{9}$$

where

$$n_{up} = \frac{\varepsilon_\infty \hbar\Omega\cos\theta(\omega_l - \omega_t)q}{2\pi e^2\Gamma}. \tag{10}$$

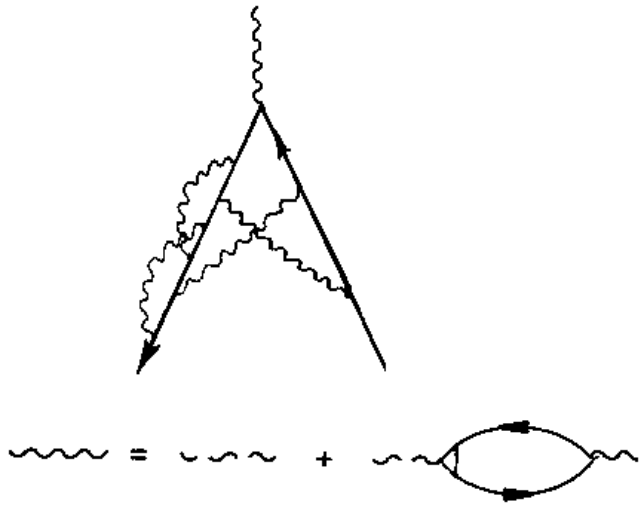

Figure 9. The typical diagram for the electron–phonon vertex, which is essential in the range $n_s/n_{up} \gg 1$. It does not allow to write an exact electron–phonon equation in closed form.

$\Gamma_{\mathcal{N}}$ should be smaller than the spacing between the Landau levels. For large concentrations n_s, in equation (9) the last term is predominant. This gives the condition

$$n_s/n_{up} \ll 1. \tag{11}$$

Here we have assumed that

$$(\omega_l - \omega_t)/\omega_l \ll 1$$

and we will neglect the terms proportional to this small parameter as compared to 1. For the estimates we will take $q = q_T \equiv \hbar^{-1}\sqrt{2mk_BT}$.

When the parameter n_s/n_{up} is of the order of 1, the MPR peaks begin to overlap and at $n_s/n_{up} > 1$ the MPR amplitude should rapidly go down as the spacing between the Landau levels becomes of the order of their width. As a more rigorous theory [5], our qualitative considerations given above permit one only to understand the origin of the sharp dependence of the MPR amplitude on n_s. One can give the following order-of-magnitude estimate $n_{up} \sim 10^{11}$ cm^{-2} as well. Here one has a many-electron problem with a strong electron–phonon interaction. In this region neither an analytical solution nor numerical simulation is possible. The point is that when one considers Feynman diagrams of higher order they acquire additional denominators of the form

$$\frac{1}{\omega - \mathcal{N}\Omega\cos\theta + i\delta}$$

that do not depend of the electron's quantum numbers. This means that every iteration of a higher order enhances the singularity of the resulting expression. The principal problem that the theory encounters is that it is impossible to write the electron–phonon vertex in a closed form for $n_s \gg n_{up}$. One example of such diagrams is given in figure 9. Note that the phonon Green functions entering such diagrams contain the exact vertices themselves.

However, one can be sure that if one follows the MPR maximum where n_s does not depend of the magnetic field B, the parameter n_s/n_{up} does not change (see equation (8)) and, as a result, the height of the MPR maximum remains constant. This is a direct consequence of the fact that the initial equation describing the problem depends only on the perpendicular component

of **B**. In the region $n_s/n_{up} > 1$, even a small variation of n_s due to the variation of B may result in a strong variation of the height of the MPR maximum. In case the height of the MPR peak depends only on the parameter n_s/n_{up} the height would remain the same irrespective of the way one changes the concentration n_s. The concentration can be changed either by magnetic field variation or by using for the measurements the samples with different concentrations determined by doping. The result has to be the same. Our samples with concentrations $n_s = 2.2$, 3.2 and 4.0×10^{11} cm^{-2} undoubtedly belong to this region. Accordingly for $n_s = 2 \times 10^{11}$ cm^{-2} the MPR amplitude is five times smaller than the maximal one and is about 1% of the background, while for $n_s = 4 \times 10^{11}$ cm^{-2} the amplitude is about 0.1%. This is approximately equal to the observed ratio signal/background in our experiment.

As shown in figure 6 of [4], the region of 2D electron concentrations n_s where MPR is observable is rather narrow, while the n_s dependence of the MPR amplitude is very sharp. We wish to check whether a relatively small electron concentration variation due to a small variation ΔB of the magnitude of the *resonant magnetic field*

$$\Delta B_{\mathcal{N}}(\theta) = B_{\mathcal{N}}(0)\left(\frac{1}{\cos\theta} - 1\right) \tag{12}$$

is sufficient to explain the decrease of the height of the MPR maximum due to the tilting of the field **B** by the angle θ. For this purpose we will consider the data of figure 6 of [4] in the interval of electron concentrations relevant for our experimental situation ($n_s \gtrsim 2\times10^{11}$ cm^{-2}). Naturally, one can expect here only an order-of-magnitude accuracy. One can see that, roughly, the MPR amplitude decreases by 25% under enhancement of the electron concentration n_s by 1%. Or, in other words, the MPR amplitude doubles under decrease of n_s by 2%. Strictly speaking, our estimates show that variation of n_s by several per cent results in a relative variation of the MPR amplitude of the order of unity. This fact, however, permits us to understand why the variation of the MPR amplitude takes place at all. This fact is in contradiction to a 2D theory for *any* region of n_s.

In figure 5 the amplitude of the second maximum at 170 K has decreased by a factor 2 at the angle $\theta_{1/2} = 25°$. The variation of the magnetic field is $\Delta B = 1.13$ T. In the perpendicular **B**, the maximum is at 11.25 T. The variation of electron concentration at this temperature is 1.5% for 1 T (figure 3). Thus the increase of the concentration is 1.7% at the angle of tilting 25°. The agreement may be considered as reasonable.

These estimates are valid for the specimens with various concentration. To check whether the angle $\theta_{1/2}$ remains the same for different ways of separating the oscillating part out of the background, we have used a more rough algorithm than above (see section 2), choosing one of the edges of the interval of the magnetic field variation towards the peak. This has widened the sides of the peak as it should do. To visualize the whole procedure we give in figure 5 only the experimental dots (not approximating them by a smooth curve). It gives practically the same value of $\theta_{1/2}$.

The rate of concentration variation as a function of magnetic field goes up with the temperature (see figure 3). If the decrease of the MPR amplitude is determined by growth of the concentration this should enhance the sharpness of the angular dependence of the MPR amplitude with the temperature. In fact this behaviour has been observed in our experiment (see figures 5, 7 and 8). Indeed, in figure 7 at $T = 190$ K for $\mathcal{N} = 2$, $\theta_{1/2} = 17°$. This corresponds to a smaller variation of the field, $\Delta B = 0.6$ T. As, however, the rate of concentration variation with B goes up with higher temperatures (it is of the order of 3% T^{-1}—see figure 3) one has in fact the same variation of the concentration $\Delta n_s/n_s = 1.8\%$. With our accuracy this coincides with the drop of the MPR amplitude under the variation of the carrier concentration in the perpendicular magnetic field **B**—see [4]. One has a decrease of the maximum by a factor of

two for an enhancement of the concentration n_s by 2%.

It may be useful to note that our reasoning can be reformulated:

(1) Consider the experimental data in the perpendicular magnetic field, i.e.:
   (a) The data of [4] on the n_s dependence of the MPR amplitude in the perpendicular magnetic field.
   (b) The n_s variation as a function of the perpendicular magnetic field determined by the Hall effect.

(2) Then one can calculate the critical angle $\theta_{1/2}$ for the tilted field and compare it with the experimental data.

We can offer the following direct experimental proof that the considered effect depends on the variation of the electron concentration n_s in the magnetic field. In the same sample, for the same variation of the MPR amplitude, it is necessary that the variation of the concentration n_s under rotation of the sample should be the same for different values of magnetic field. In other words,

$$\Delta n_s = \Delta B_{\mathcal{N}}(\theta)\frac{\partial n_s}{\partial B}$$

should be $\mathcal{N}$-independent. According to figure 3, in the interval of the field variation 4–12 T the concentration is within our accuracy a linear function of the field B. In other words, for $\mathcal{N} = 2$ and 3, $\Delta B_{\mathcal{N}}(\theta_{1/2}^{(\mathcal{N})})$ should be $\mathcal{N}$-independent. Then, according to equation (12), we have for example for $\mathcal{N} = 2$ and 3

$$\cos\theta_{1/2}^{(3)} = \frac{B_3(0)}{B_3(0) + B_2(0)(1/\cos\theta_{1/2}^{(2)} - 1)}. \tag{13}$$

The sharper the peak, the more sensitive is equation (13) to the variation of the angles $\theta_{1/2}$. As the peaks become more narrow with the temperature, we have chosen $T = 230$ K. Then for $\mathcal{N} = 2$ we have $\theta_{1/2}^{(2)} = 12°$. As $B_2(0) = 11.25$ T, $B_3(0) = 7.5$ T we get $\theta_{1/2}^{(3)} = 15°$, which is in good agreement with the experimental value (see figure 8).

4. Conclusion

In summary, we have investigated magnetophonon resonance in a tilted magnetic field measuring also the 2D electron concentration of the same samples. Analysing the experimental data we have arrived at the following conclusions. The sharp angular dependence of the MPR maxima on θ is a manifestation of a very sharp concentration dependence of the MPR amplitude in the perpendicular magnetic field. The reason as to why the 2D concentration of the carriers can be enhanced is, as we understand, the following. Due to the tilting of the magnetic field the MPR maximum is shifted towards the strong magnetic fields (see equation (1) and figure 1 which agrees with the data of [4]). The shift is comparatively small (of the order of 1 T) and at high temperatures brings about a comparatively small concentration variation (of the order of several per cent). However, due to a very sharp concentration dependence of the MPR amplitude, this is sufficient for a decrease of the amplitude by several times in the relevant concentration interval [11].

It would be very interesting to investigate in future the MPR in quantum wells of various compositions. It is also desirable to make a systematic investigation of the MPR in nanostructures of different forms, such as quantum wires (see, for instance [11]) as well as to take into consideration the polaron effect [12].

Thus the principal conclusion of the paper can be formulated as follows. The angular dependence of the MPR amplitudes as well as the decrease of the resonance widths with the temperature is a manifestation of dependence of n_s on the total magnetic field B (observed explicitly in the present paper). This statement permits us to relate three seemingly different groups of experiments performed in different laboratories.

(1) A sharp decrease of the MPR amplitude in a perpendicular magnetic field as a function of growing n_s with a steep angular dependence of the MPR amplitude under the tilting of the magnetic field.
(2) A narrowing of the angular dependence of the MPR peaks as a function of rising temperature with the enhancement of the rate of variation of n_s as a function of B. When the temperature goes up the function $n_s(B)$ becomes more sharp.
(3) The characteristic width of the MPR for different $\mathcal{N}$ with the rate of variation of n_s as a function of B.

We wish to emphasize that the dependence $n_s(B, T)$ has not been an adjustable function. Rather it has been extracted from the Hall effect measured on the same samples. The variation of the electron concentration in quantum wells as a function of magnetic field may be also important for analysis of other transport phenomena in quantum wells.

Acknowledgments

This work is supported by the Wihuri Foundation, Finland. VVA and VLG also acknowledge partial support for this work by the Russian National Fund of Fundamental Research (Grant No 03-02-17638).

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